Abstract Algebra I
Problem Set 3: Homomorphisms

Example style questions are marked with (*).

<u>Homomorphisms</u>

Question 1. Let $\varphi : G_1 \to G_2$ and $\psi : G_2 \to G_3$ be isomorphisms. Show $G_1 \cong G_3$.

Question 2. Prove that every infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$.

Question 3. (*) Let $\varphi : G \to F$ be an isomorphism. Prove:

- $\varphi(e_g) - e_F$; and
- $\varphi(g^{-1}) = \big(\varphi(g)\big)^{-1}$ for all $g \in G$.

In each case determine whether φ needs to be an isomorphism, i.e. whether the two statements are true for all homomorphisms.

Question 4. (*) Let a be an element of a group G. Consider the mapping $\varphi_a : G \to G$ defined by $\varphi_a(g) = ag$ for all $g \in G$.

- Prove that φ_a is a permutation of the set of elements of G, i.e. that φ_a is a bijection.
- Consider the set $\{\varphi_a \,|\, a \in G\}$ of all permutations defined in the above way. Prove that this set forms a group with standard composition of mappings.
- Denote the group defined in the previous part by Φ. Prove that $G \cong \Phi$.